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Fermilab

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S. Ohnuma

An Estimate of the Vertical Dispersion Function

During the doubler review meeting on June 14th, a question was raised on the expected vertical dispersion function arising from error fields. Three major causes for the vertical dispersion are considered here:

1. Skew quadrupole field of dipoles.
2. Rotation of quadrupoles.
3. Vertical closed orbit distortion coupled with normal sextupole field.

In general, if the skew quadrupole gradient is B', the vertical dispersion function is

$$\eta_y(s) = \frac{\sqrt{\beta_y(s)}}{2\sin(\pi\nu_y)} \int \sqrt{\beta_y}\,\eta_x(B'/B\rho)\cos(\pi\nu_y - |\Delta\psi_y|)\,dz$$

where η_x is the horizontal dispersion function and $\Delta\psi_y$ is the phase advance from <u>z</u> to <u>s</u>. The integral is for the entire ring. One is interested in the amplitude of the dispersion, that is, the quantity which is a measure of the effective emittance growth. The amplitude will be called $\hat{\eta}_y$ here and the corresponding emittance is $(\hat{\eta}_y\Delta p/p)^2/\beta_y$.

The probability distribution of the expected value of $\hat{\eta}_y$ depends on the importance of harmonic components,[1]

$$f_k = \frac{1}{2\pi}\int\sqrt{\beta_y^3}\,\eta_x(B'/B\rho)\,e^{-ik\phi}\,d\phi$$

where ϕ is the phase advance divided by ν_y. For example, when the tune is very close to an integer N, the contribution from f_N is dominant and the rms value of $\hat{\eta}_y$ is

$$\langle \hat{\eta}_y \rangle = \frac{\sqrt{\beta_y}}{2|\sin(\pi\nu_y)|} \langle B'/B\rho \rangle \sqrt{\Sigma \beta_y \eta_x^2 \ell^2}$$

where ℓ is the length of each magnet and the summation is over all magnets with rms error field $\langle B' \rangle$. In most cases, several harmonic components contribute significantly and one must multiply the above expression by a certain factor. CERN "Yellow Book"[2] gives 1.73 and the estimate by Gluckstern[3] is 1.57 when the fractional part of the tune is 0.4. In this note, the factor 2 is used. The quantity $\hat{\eta}_y$ is then expected to be less than $\langle \hat{\eta}_y \rangle$ with approximately 85% probability.

1. <u>Skew Quadrupole Component of Dipoles</u>

$|B'/B| < 4\times10^{-4}$/inch, uniform distribution

$\nu_y = 19.4$, $\ell/\rho = 2\pi/774$, $\sqrt{\Sigma \beta_y \eta_x^2} = 613.8\ m^{3/2}$

$$\langle \hat{\eta}_y \rangle / \sqrt{\beta_y} = 2 \times 0.0238\ m^{1/2} = 0.0476\ m^{1/2}$$

At $\beta_y = 100m$, $\langle \hat{\eta}_y \rangle = 0.48$ m.

2. <u>Rotation of Quadrupoles</u>

Assume the rms value of roll angle to be 1 mrad.

B'_o (normal quadrupole gradient) = 760 kG/m at 1,000 GeV/c.

$\langle B'/B\rho \rangle = 2\times0.001\times(760/33356) = 4.56\times10^{-5}\ m^{-2}$

$\sqrt{\Sigma \beta_y \eta_x^2 \ell^2} = 566.1\ m^{5/2}$ (summation over all quadrupoles)

$$\langle \hat{\eta}_y \rangle / \sqrt{\beta_y} = 2\times0.0136\ m^{1/2} = 0.0272\ m^{1/2}$$

At $\beta_y = 100m$, $\langle \hat{\eta}_y \rangle = 0.27$ m.

3. Sextupole Field and the Vertical Orbit Displacement

The vertical orbit displacement Δy coupled with the normal sextupole field produces skew quadrupole field, $B' = B''(\Delta y)$.

Take $\langle\Delta y\rangle \equiv \sqrt{\beta_y}\,\Delta$ and $|(^1/_2 B''/B)| < 6\times10^{-4}/\mathrm{inch}^2$.

$$\sqrt{\Sigma\beta_y^2\eta_x^2} = 4.71\times10^3\ \mathrm{m}^2$$

$$\langle\hat{\eta}_y\rangle/\sqrt{\beta_y} = 2\times21.6\Delta = 43.2\Delta \qquad (\Delta \text{ in } \mathrm{m}^{1/2})$$

Assume $\Delta y = 5$ mm at $\beta_y = 100$ m, that is, $\Delta = 5\times10^{-4}\ \mathrm{m}^{1/2}$,

$$\langle\hat{\eta}_y\rangle/\sqrt{\beta_y} = 0.0216\ \mathrm{m}^{1/2}$$

At $\beta_y = 100\mathrm{m}$, $\langle\hat{\eta}_y\rangle = 0.22$ m.

If all three errors are combined quadratically, we get

$$\langle\hat{\eta}_y\rangle = 0.059\sqrt{\beta_y}\,. \qquad (\eta_y \text{ and } \beta_y \text{ in meters})$$

1. Courant-Snyder, p. 18.
2. C. Bovet, et al., CERN/MPS-SI/Int. DL/70/4, 23 April, 1970, p.24.
3. R. L. Gluckstern, Particle Accelerators, 8 (1978), p.203.